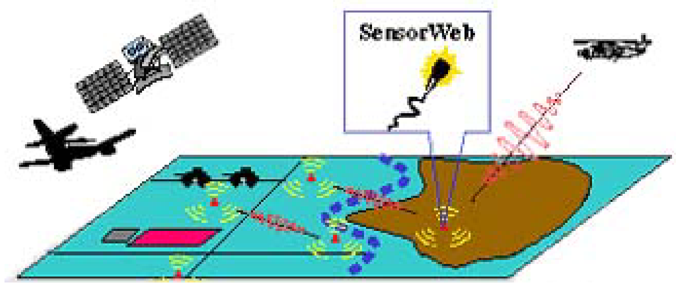

Scalability and capacity of networks with large numbers of nodes

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SensorWeb MURI Review Meeting, June 18, 2001

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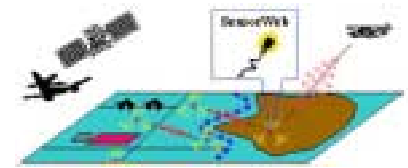
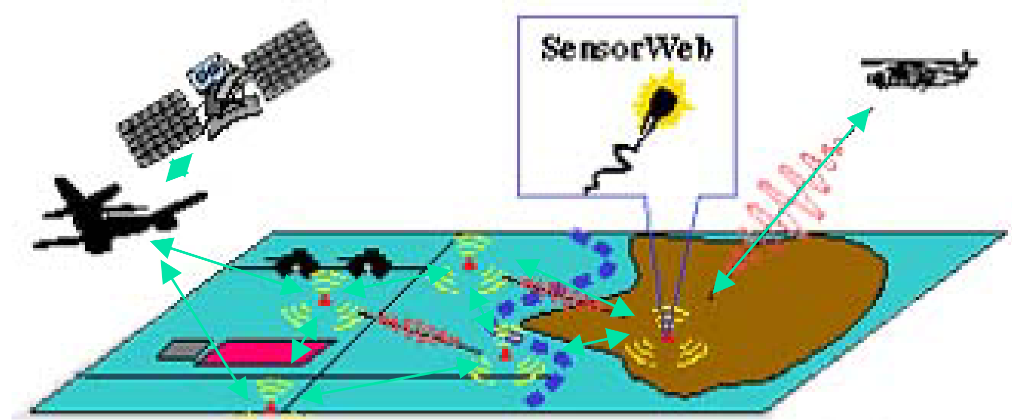
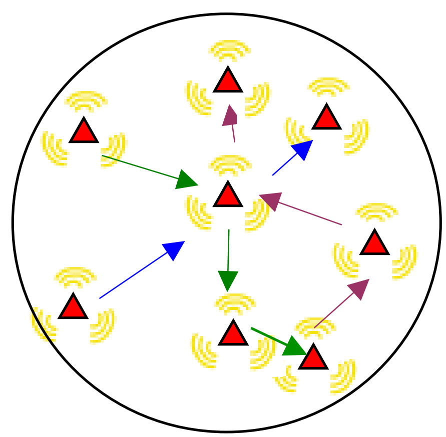
Sensor web networks



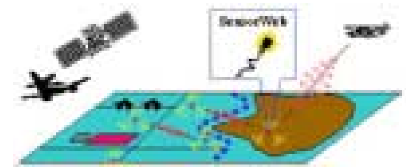
- Networks with large numbers of sensors
  - Potentially large number of information gathering nodes
  - Connected by wireless medium
  - Possibly low power nodes



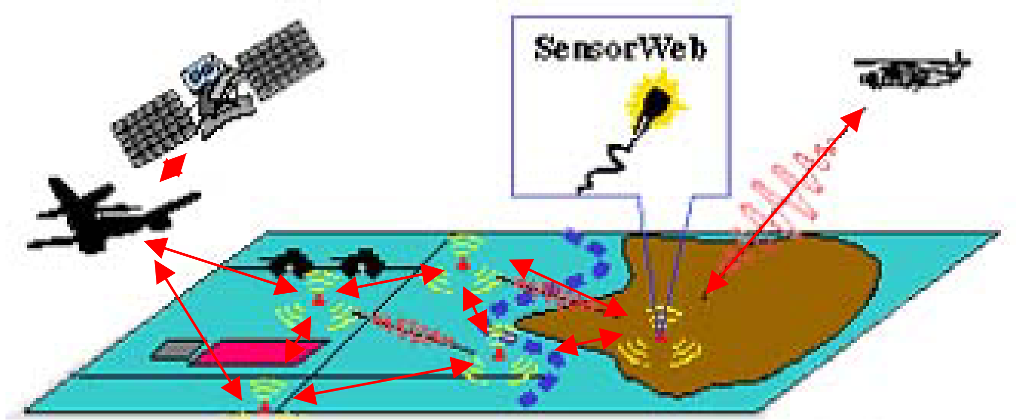


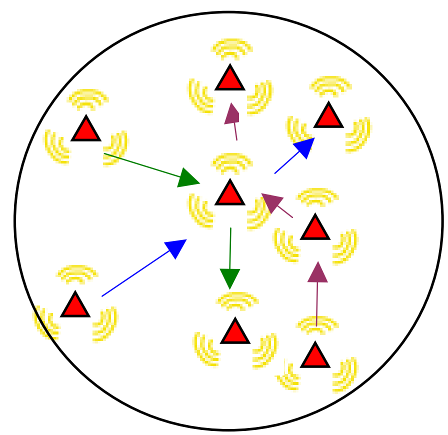
- IT-3: Wireless networks, network communication and information theory
- RCA 2&3: Fundamental limits on fusion, Network Info Theory, Tradeoffs in local vs. global processing

Scalability and capacity issues

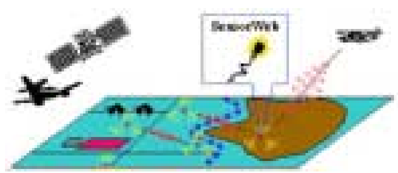


- Issues
  - How much traffic can they carry?
    - A Technological model
  - Experimental scaling law for current technology
  - How does power usage for communication scale?
  - How does range of nodes scale?
  - What is an architecture for operating them?
  - How does one increase the information carrying capacity with current technology?
  - What is ultimately possible?
    - A large scale information theory for networks

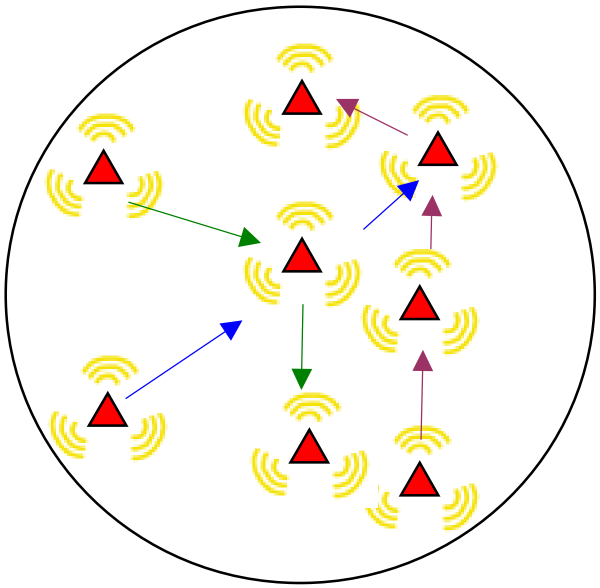




How much traffic can wireless networks carry?

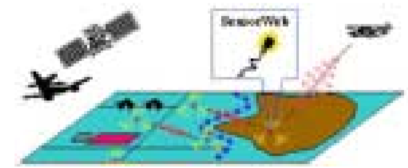


- Model
  - Disk of area A sq.m
  - n nodes
  - Each can transmit at W bits/sec



- Shared wireless channel: Interference between transmissions
  - Protocol Model
  - Physical Model (Signal-to-Interference Ratio)
- What can they provide?
  - Throughput for each node: Measured in Bits/Sec
  - Traffic carrying capacity of entire network: Measured in Bit-Meters/Sec
  - Scaling with the number of nodes n

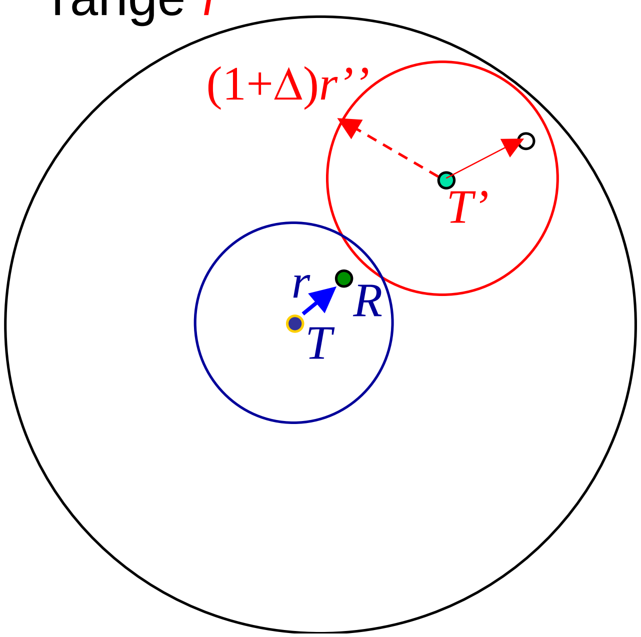
Protocol and Physical Models



- <u>Protocol Model</u>

  Receiver R should be

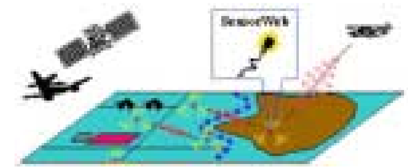
  - within range r of its transmitter T
  - outside footprint $(1+\Delta)r'$ of any other transmitter T' using range r'



- <u>Physical Model (SIR Model)</u>

$$\text{SIR Ratio} = \frac{P_i r_i^{-\alpha}}{N + \sum_{j \neq i} P_j r_j^{-\alpha}} \geq \beta$$

Optimally located and operated networks



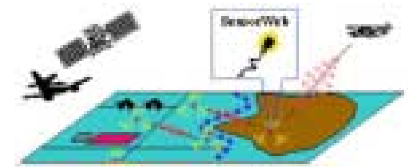
- Optimal network
  - Optimally located nodes, destinations, demands for OD-pairs
  - Optimal spatial and temporal scheduling, routes, ranges for each transmission

- Protocol Model: Network can transport $\Theta\left(W\sqrt{An}\right)$ bit-meters/sec

$$\frac{W}{1+2\Delta}\frac{n}{\sqrt{n}+\sqrt{8\pi}} \leq \text{Best case capacity for Protocol Model} \leq \sqrt{\frac{8}{\pi}}\frac{W}{\Delta}\sqrt{n} \quad \text{bit-meters/sec}$$

- If equitably divided, each node can send $\Theta\left(W\sqrt{\frac{A}{n}}\right)$ bit-meters/sec
  - Transport capacity does *not* scale linearly
    - Unicast model

Best possible scenario



- Physical Model:

$$\frac{1}{\left(16\beta\left(2^{\alpha/2}+\frac{6^{\alpha-2}}{\alpha-2}\right)\right)^{1/\alpha}}\frac{Wn}{\sqrt{n}+\sqrt{8\pi}} \le$$

Best case capacity for Physical Model bit-meters/sec

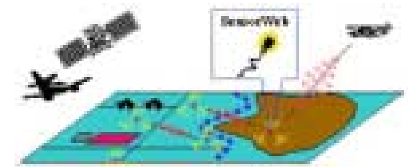
$$\le \left(\frac{2\beta+2}{\beta}\right)^{1/\alpha}\frac{W}{\sqrt{\pi}}\, n^{(\alpha-1)/\alpha}$$

bit-meters/sec

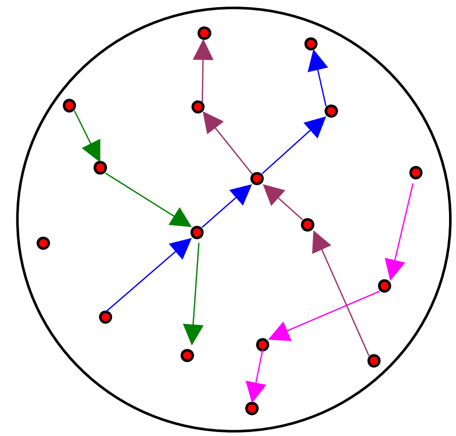
Can be sharpened to $\Theta(\sqrt{n})$ if $P_{max}/P_{min} < \beta$

- Suggests better capacity with greater path loss
- Upper bound $O\left(n^{(\alpha-1)/\alpha}\right)$ needs sharpening

Randomly formed networks



- *n* nodes randomly located
  - Each node chooses random destination
  - Equal throughput λ bits/sec for all OD pairs
  - Each node chooses same range *r*



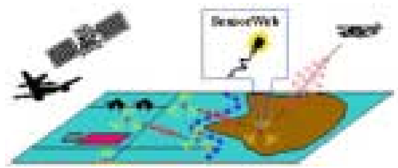
- Best choice of spatio-temporal scheduling, ranges and routes
- Each node can send $\Theta\left(\frac{1}{\sqrt{n \log n}}\right)$ bits/sec { Random networks are nearly best
  Also: Lowest common range for connectivity is optimal
- Definition of capacity

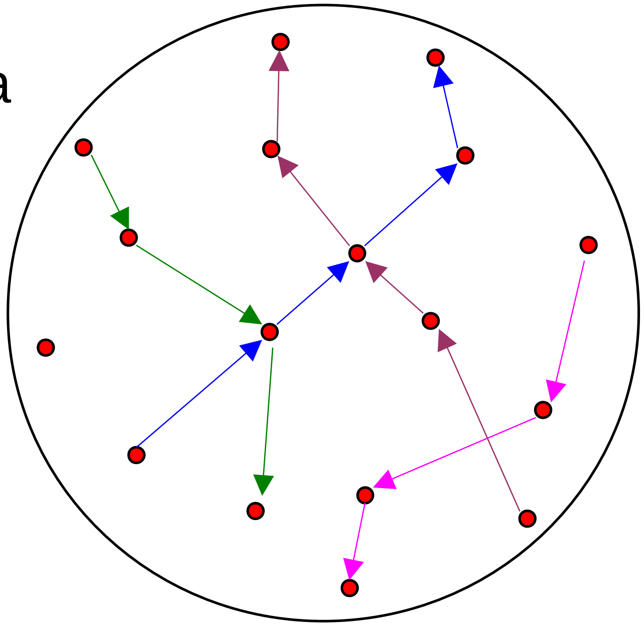
$$\lim_{n\to\infty} \Pr(\lambda(n) = \frac{c}{\sqrt{n \log n}} \text{ is feasible}) = 1, \text{ and}$$

$$\lim_{n\to\infty} \mathrm{P}(\lambda(n) = \frac{c'}{\sqrt{n \log n}} \text{ is feasible}) = 0$$

{ Sharp cutoff phenomenon

Physical model: Random network

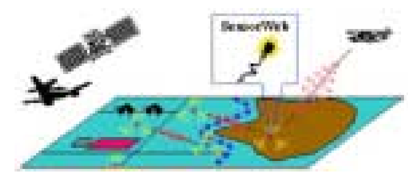




- *n* nodes randomly located in disk of unit area
  - Each node chooses random destination
  - Equal throughput λ bits/sec for all OD pairs
  - Each node chooses same power level *P*
- Theorem
  - With best choice of routes, hops, spatio-temporal scheduling

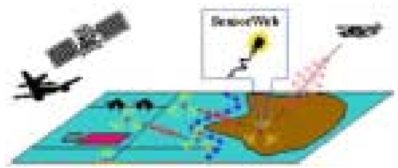
$$\Theta\left(\frac{1}{\sqrt{n\log n}}\right) \leq \lambda(n) \leq \Theta\left(\frac{1}{\sqrt{n}}\right) \text{ bits/sec}$$

Implications for Sensor Web design



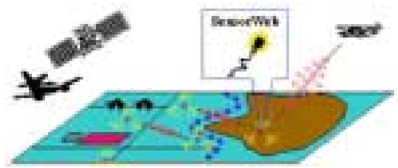
- Transport capacity is constrained to $\Theta(W\sqrt{An})$ bit-meters/sec
- So design network with either
  - Few nodes
    - Small n
  - Or scaled down bandwidth
    - Small λ
  - Or support mainly nearest neighbor communications
    - Small "meters"
    - Nearest neighbor is $\Theta\left(\frac{1}{\sqrt{n}}\right)$ meters away
    - Architecture for data fusion in sensor networks

Implications for Sensor Web design



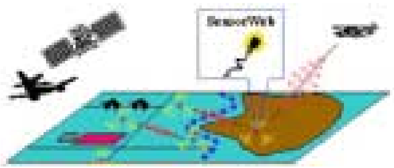
- Architecture for providing optimal capacity
  - Group nodes into cells of size O(log *n*)
  - One node in each cell serving as relay
  - Strategy for organizing sensor webs for communication purposes
- Power consumption
  - Percentage of their time that nodes are busy communicating is $\Theta\left(\frac{1}{\log n}\right)$
  - So asymptotically, power consumption for communication is not the bottleneck
- Range of transmissions
  - Scaled length of hops is $\Theta\left(\sqrt{\frac{\log n}{n}}\right)$
  - Lower power needed when there are more nodes
- Splitting into several sub-channels (TDMA, FDMA, CDMA) does not help in increasing capacity in these models

Implications for Sensor Web design

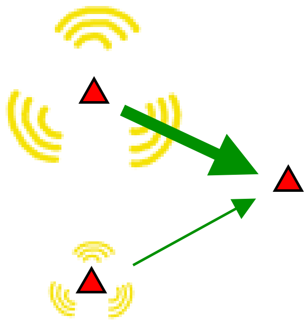


- How to increase capacity under this model?
  - Add kn randomly placed relay nodes
  - These additional nodes don't spawn any traffic of their own
  - They are merely there to help other nodes by relaying their messages
  - Then the capacity is increased by a factor $\sqrt{k}$
- Directed transmissions will help
  - The smaller the wireless footprint the better
  - Space is a valuable resource
  - However it changes only the constant, not the growth rate

What are the ultimate limits?



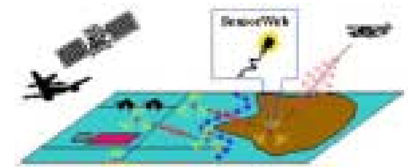
- Above model of technology is not most general one
  - Interference is not interference.



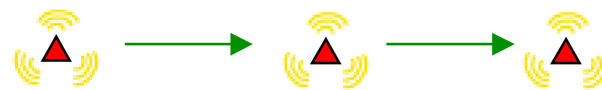
  - Receiver can first decode loud signal perfectly
  - Then subtract the loud signal
  - Then decode the soft signal perfectly

  - So excessive interference can be very good
- Need an information theoretic foundation for large networks

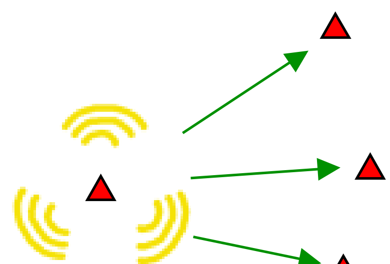
Elements of cooperation in networks



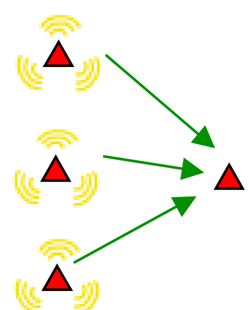
- Nodes can cooperate in many ways in networks
  - Relaying



  - Broadcasting

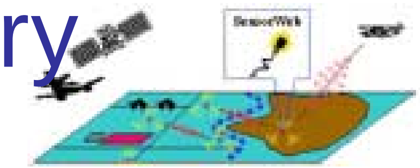


  - Multiple access

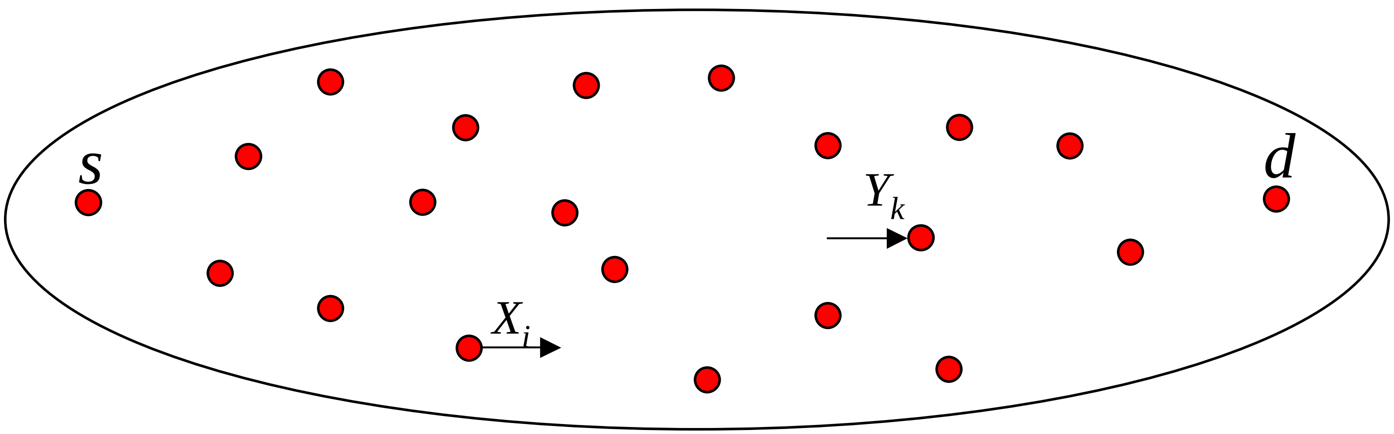


  - ....
- How to weave a scalable information theory with multiple modalities of cooperation?

Towards an information theory for large networks



- Consider any set of n nodes



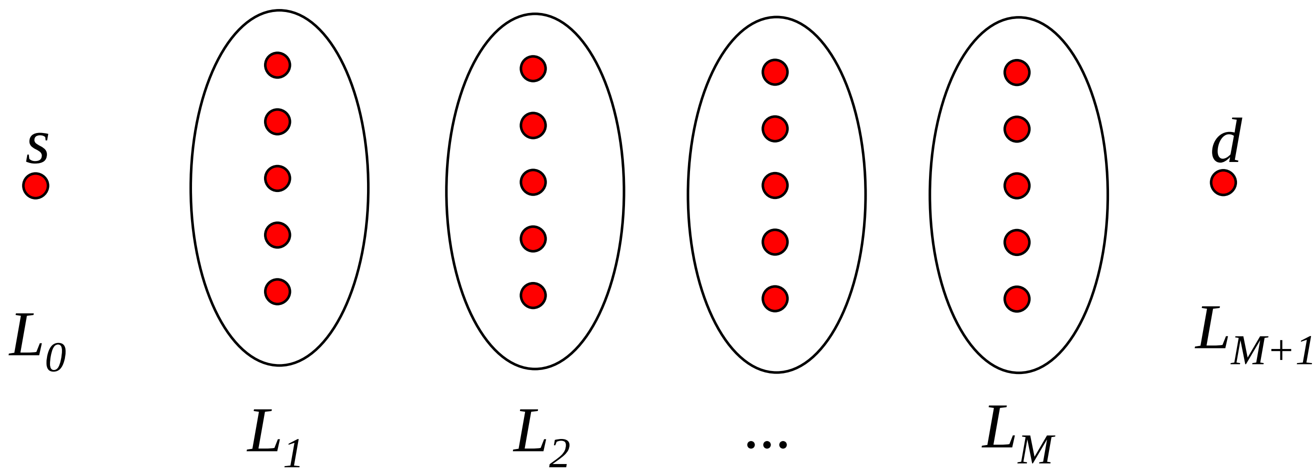
- General vector, memoryless, discrete channel $p(Y_1, \ldots, Y_n \mid X_1, \ldots, X_n)$

- How much information can be carried from s to d?

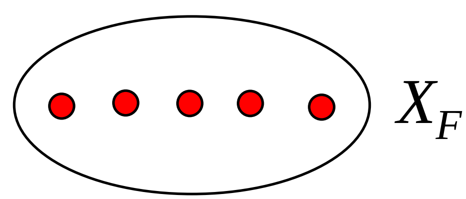
Feedforward flow graph

- Group nodes into levels

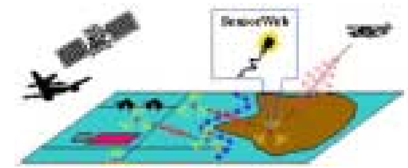
- Form a feedforward graph



- Some nodes X_F can be left out



Achievable rate for arbitrarily large networks



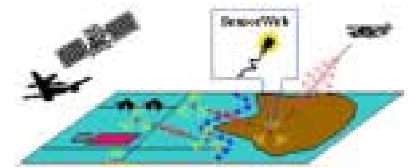
- Let $R_M, R_{M-1}, \ldots, R_1, R_0$ be defined by

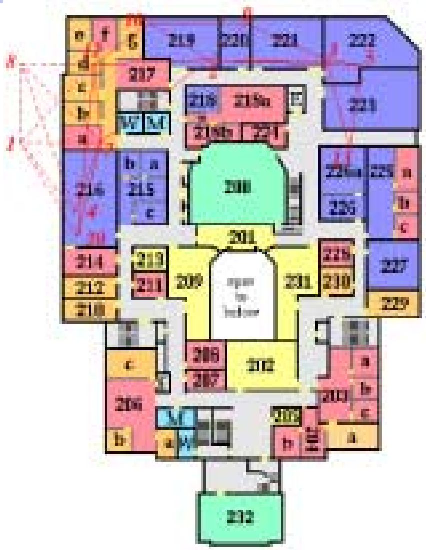
$$R_M = I(X_M; Y_{M+1} \mid x_F)$$

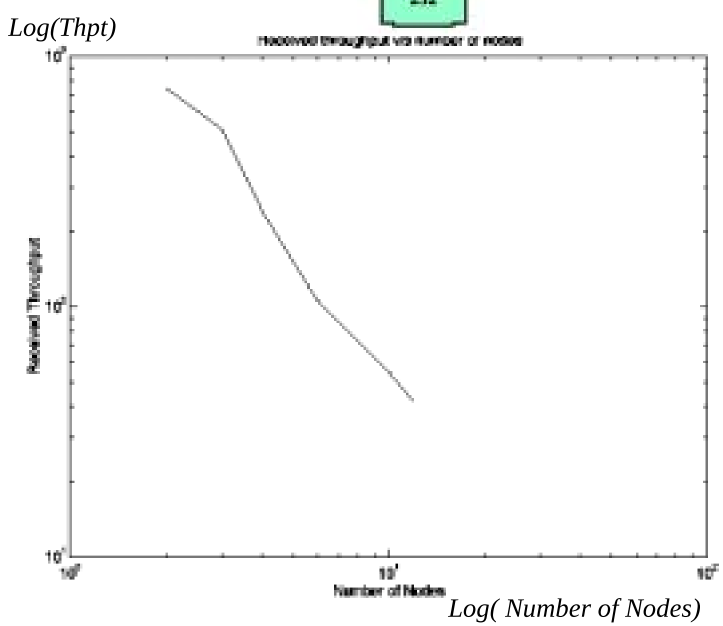
$$R_m = Min\{\underset{j}{Min}\, I(X_m; Y_{m+1,j} \mid X_{m+1}, \ldots, X_M, x_F),$$
$$R_{m+1} + \underset{k \geq 2}{Min}\, \underset{j}{Min}\, I(X_m; Y_{m+k,j} \mid X_{m+1}, \ldots, X_M, x_F)\}$$

- Then, any rate less than R_0 is feasible
  - Includes many results in, e.g., relay channel, multiple access channel, etc, as special cases

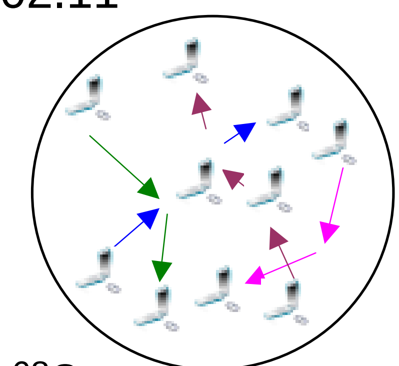
Experimental scaling law





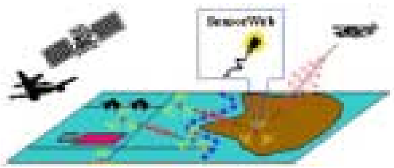


- Throughput = $2.6/n^{1.68}$Mbps per node
  - No mobility
  - No routing protocol overhead
    - Routing tables hardwired
  - No TCP overhead
    - UDP
  - IEEE 802.11



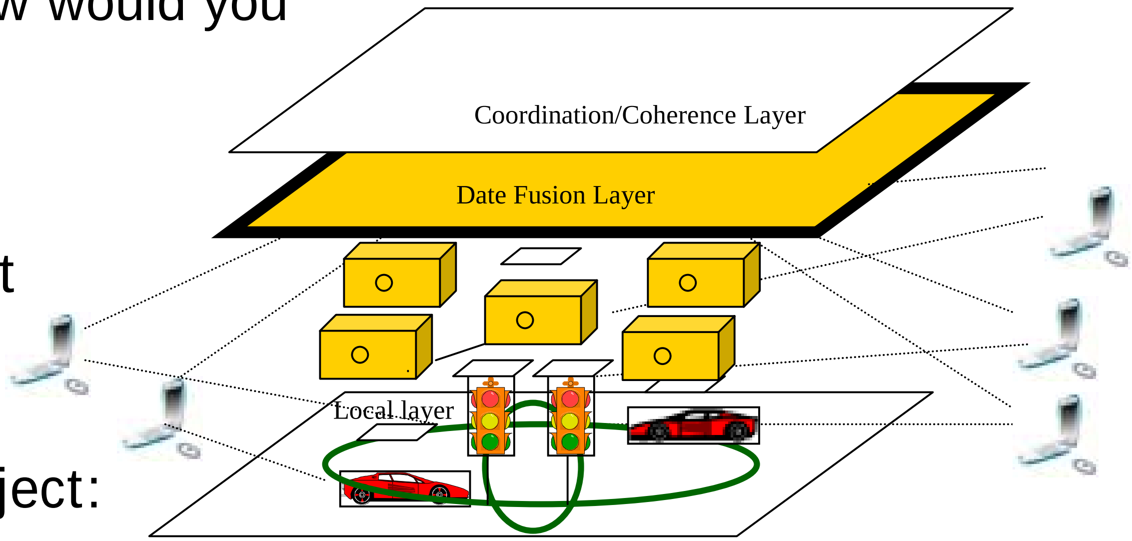
- Why $1/n^{1.68}$?
  - Much worse than optimal capacity = $c/n^{1/2}$
  - Worse even than 1/n timesharing
  - Perhaps overhead of MAC layer?

An experimental testbed for networking sensors



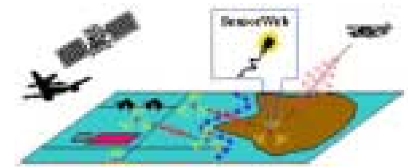
- Next step in IT revolution: Convergence of communication, computing, and control
- Sensors and actuators galore communicating over wireless and interacting with physical world
- Issue: How do we organize such distributed real-time systems?
  - Eg. If traffic lights and cars and sensors can talk to each other, how would you architect the system?

What are the right abstractions?



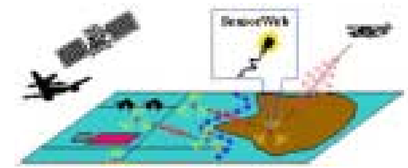
- A testbed for convergence at Univ of Illinois
- Layer of interest for this project:
  - Sensing, Networking, Data Fusion layers

U. S. Army interactions



- Panel Member, Triennial Research Strategy Planning Workshop U.S. Army Research Office, Computing and Information Sciences Division, Charleston, SC, Jan 3-5, 2001.
- Board of Visitors of the U.S. Army Research Office, 6.1 Mathematical Sciences Program Review, May 21, 2001, Research Triangle Park

Other events



- Plenary Talk, SIAM Annual Meeting, July 9-13, 2001, San Diego
- Panel, Future Directions in Control and Dynamical Systems, June 16-17, 2000.
- Conference on Stochastic Networks, June 19-24, 2000, University of Wisconsin, Madison
- NSF/ONR Workshop on Cross-Layer Design in Adaptive Ad Hoc Networks: From Signal Processing to Global Networking, May 31-June 1, 2001, Cornell University
- Symposium on Complex Systems Modeling and Optimization in the Information Age To Celebrate 45 Years of Outstanding Contribution of Prof. Yu-Chi "Larry" Ho, June 23-24, 2001, Harvard University
- Chair, Workshop on Wireless Networks, Institute for Mathematics and its Applications, Minneapolis, August 8-10, 2001.
- 10th Mediterranean Conference on Control and Automation, Lisbon, Portugal, July 9-13, 2002.